## CBGS SCHIMME

USN


Fifth Semester B.E. Degree Examination, July/August 2021 Automata Theory and Computability

Time: 3 hrs .

## Note: Answer any FIVE full questions.

1 a. Define the following terms with examples:
(i) Alphabet
(ii) Strings
(iii) Kleene's closure
(iv) Languages
(v) Concatenation
(05 Marks)
b. Draw a DFA to accept the following languages.
(i) $\mathrm{L}=\left\{\mathrm{w} \in\{\mathrm{a}-\mathrm{z}\}^{*}\right.$, all five vowels $\mathrm{a}, \mathrm{e}, \mathrm{i}$, o and $u$ occur in w in alphabetical order $\}$
(ii) $\mathrm{L}=\left\{\mathrm{w} \in\{\mathrm{a}, \mathrm{b}\}^{*}\right.$, set of all strings containing the substring "aab" $\}$
(06 Marks)
c. Convert the following $\in-$ NFA to its equivalent DFA. [Refer Fig.Q1(c)]


Fig.Q1(c)

2 a. Obtain a DFA to accept the following language.

$$
\mathrm{L}=\left\{\mathrm{w} \in\{\mathrm{a}, \mathrm{~b}\}^{*}, \quad \mathrm{~N}_{\mathrm{a}}(\mathrm{w}) \bmod 5=0 \text { and } \mathrm{N}_{\mathrm{b}}(\mathrm{w}) \bmod 3=0\right\}
$$

b. Give the differences between DFA, NFA and $\in$-NFA.
c. Minimize the following DFSM. [Refer Fig.Q2(c)]


Fig.Q2(c)
(09 Marks)
3 a. Obtain a regular expression for each of the following languages:
(i) $\mathrm{L}=\left\{\mathrm{w} \mid \mathrm{w} \in\{\mathrm{a}, \mathrm{b}\}^{*}\right.$ with atleast three consecutive zero's $\}$
(ii) $\mathrm{L}=\left\{\mathrm{w} \in\{\mathrm{a}, \mathrm{b}\}^{*}\right.$ set of all strings starting with a and ending with b$\}$
(iii) $\mathrm{L}=\left\{\mathrm{w} \mid \mathrm{w} \in\{\mathrm{a}, \mathrm{b}\}^{*}\right.$ whose second symbol from the right end is ' a ' $\}$
b. Obtain the regular expression for the following FSM using Kleene's theorem.


Fig.Q3(b)
(10 Marks)

4 a. Show that the following languages are not regular:
(i) $\mathrm{L}=\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{\mathrm{n}} \mid \mathrm{n} \geq 0\right\}$
(ii) $L=\left\{1^{\mathrm{p}} \mid \mathrm{p}\right.$ is prime $\}$
(08 Marks)
b. Simplify the following regular expression $\left(\left(a^{*} \cup \phi\right)^{*} \cup a a\right)(b \cup b b)^{*} b^{*}\left((a \cup b)^{*} b^{*} \cup a b\right)^{*}$
(06 Marks)
c. If $\mathrm{L}_{1}$ and $\mathrm{L}_{2}$ are regular languages, then prove that $\mathrm{L}_{1} \cup \mathrm{~L}_{2}, \mathrm{~L}_{1} \cdot \mathrm{~L}_{2}$ and $\mathrm{L}_{1}^{*}$ are regular languages.
(06 Marks)

5 a. Obtain a grammar to generate each of the following languages:
(i) $\mathrm{L}=\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{2 \mathrm{n}}: \mathrm{n} \geq 0\right\}$
(ii) $L=\left\{\widehat{w} w^{R}\right.$ where $\left.w \in\{a, b\}^{*}\right\}$
(05 Marks)
b. If the following grammar ambiguous?
$\mathrm{S} \rightarrow \mathrm{aS} \mid \mathrm{X}$
$X \rightarrow a X \mid a$
(05 Marks)
c. Convert the following grammar to Chomsky Normal Form (CNF).
$\mathrm{S} \rightarrow \mathrm{aACa}$
$\mathrm{A} \rightarrow \mathrm{B} \mid \mathrm{a}$
$\mathrm{B} \rightarrow \mathrm{C} \mid \mathrm{c}$
$\mathrm{C} \rightarrow \mathrm{cC} \mid \epsilon$
(10 Marks)

6 a. Define PDA and obtain a PDA to accept a string of balanced parenthesis.
(04 Marks)
b. Construct a PDA to accept the language $L=\left\{w^{2} w^{R} / w \in\{a, b\}^{*}\right\}$. Draw the graphical representation of this PDA. Show the moves made by this PDA for the string "abCba"
(10 Marks)
c. Convert the following grammar into equivalent PDA.
$\mathrm{E} \rightarrow \mathrm{E}+\mathrm{T}$
$\mathrm{E} \leftrightarrows \mathrm{T}$
$\mathrm{T} \rightarrow \mathrm{T} * \mathrm{~F}$
$\mathrm{T} \rightarrow \mathrm{F}$
$\mathrm{F} \rightarrow$ (E)
$\mathrm{F} \rightarrow$ id
(06 Marks)

7 a. If $L_{1}$ and $L_{2}$ are Context Free Languages (CFL's), then prove $L_{1} \cup L_{2}, L_{1} \cdot L_{2}$ and $L_{1}^{*}$ are context free languages.
(05 Marks)
b. State and prove pumping lemma for context free languages and show that $L=\left\{a^{n} b^{n} c^{n} \mid n \geq 0\right\}$ is not context free.
(10 Marks)
c. Explain with neat diagram the working of turing machine model.

8 a. Explain with neat diagram, the model of Linear Bounded Automata (LBA).
(06 Marks)
b. Design a TM (Turing Machine) that accepts $L=\left\{0^{n} 1^{\mathrm{n}} \mid \mathrm{n} \geq 1\right\}$.
c. Consider the turing description given in the following table. Draw the computation sequence of the input string " 00 ".
(08 Marks)

| Present State | Tape symbols |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | b | 0 | 1 |  |
| $\rightarrow \mathrm{q}_{1}$ | $1 \mathrm{Lq}_{2}$ | $0 \mathrm{R} \mathrm{q}_{1}$ |  |  |
| $\mathrm{q}_{2}$ | $\mathrm{bRq}_{3}$ | $0 \mathrm{Lq}_{2}$ | $1 \mathrm{Lq}_{2}$ |  |
| $\mathrm{q}_{3}$ | - | $\mathrm{bR} \mathrm{q}_{4}$ | $\mathrm{~b} \mathrm{R}_{5}$ |  |
| $\mathrm{q}_{4}$ | $0 \mathrm{Rq}_{5}$ | $0 \mathrm{R} \mathrm{q}_{4}$ | $1 \mathrm{Rq}_{4}$ |  |
| $\mathrm{q}_{5}$ | $0 \mathrm{Lq}_{2}$ |  |  |  |

9 a. M is a turing machine represented by the transition diagram. Obtain the computation sequence of M for processing the input string " 0011 ". \{Refer Fig.Q9(a)]


Fig.Q9(a)
(06 Marks)
b. Design a Turing Machine (TM) to recognize all strings consisting of an even number of 1 's.
c. Design a Turing Machine (TM) to recognize the language. $L=\left\{1^{n} 2^{n} 3^{n} \mid n \geq 1\right\}$

10 Write short notes on:
a. Decidable and undecidable languages
b. Halting problem of TM
c. Post-correspondence problem
d. Church-Turing thesis

